# MATHS WITH MANIPULATIVES: BEADSTRINGS 

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Beadstrings come in different lengths and slightly different configurations. Beads are threaded onto a string in particular configurations - usually five beads of one colour and five of another colour for shorter beadstrings (see Figure 1, right), and ten of each colour for longer beadstrings (see Figure 2, right).

Similar manipulatives include Rekenreks and the Slavonic Abacus. Rekenreks include two rows of beads (See Figure 3). The beads are presented in a five of one colour, five of another colour configuration. The Slavonic Abacus (sometimes referred to as a Bead Frame) includes ten rows of ten beads (see Figure 4). As with the Rekenrek, these are presented in a five of one colour, five of another colour configuration.


Figure 3. Rekenrek.


Figure 4. Slavonic abacus.
An important link can be made between the five-five pattern and the number of digits on each hand (see Figure 5).



Figure 1. 5 of each colour beadstring.

Figure 2. 10 of each colour beadstring.
In this article we will focus on the 20 beadstring, 100 beadstring and 120 beadstring. Typically, beastrings have been used to support counting, but they can do so much more. Below we will explore contexts involving counting, addition, fractions, decimals and percentages.

## COUNTING

Beadstrings are an excellent way to support counting. Children can:

- count forward in ones, $1,2,3 \ldots$ etc.
- count forwards from any point, 5 , 6, 7, 8
- count backwards
- skip count e.g., $5,10,15,20 \ldots$
- count in multiples e.g., $3,6,9$ and so on. In this example each bead would represent a value of 3. Questions may be posed such as on a twentybeadstring, what number will you say when you move the last bead if you are counting in threes? (60).
- count in fractions, one-fourth, twofourths, three-fourths. On a twentybeadstring, what number will you say when you move the last bead if you are counting in fourths? (5)

There are many variations to the above, including clapping and moving the appropriate number of beads according to the number of claps.

## COMPARING AND ORDERING

The length of two beadstrings may be compared showing which one is longer, that is, contains more beads or less beads (see Figure 6)


Figure 6: Comparing beadstrings.
The length of three beadstrings may be ordered from the smallest (shortest) to the largest (longest) (see Figure 7)


Figure 7: Ordering beadstrings.

## A MODEL OF PART PART WHOLE THINKING

The ten beadstring may be used to develop part part whole understanding (see Figure 8). Later children may be encouraged to look for more than two parts. For example, 3, 3 and 4.


Figure 8: Representing Part Part Whole on a beadstring

The ten beadstring may be partitioned into two parts and the parts represented in a number sentence. For example, 7 and 3 is $10(7+3=10)$. Later a number sentence can be written and the beads moved to show what the number sentence says.

Finally, the beadstring may be spilt into two parts and one part covered. This extends the thinking to finding missing parts, given a whole and one part.

Figure 5. Hands and five-five pattern.

## ADDITION

The beadstring may be used to show the commutative property of addition, sometimes referred to as the 'turnaround' facts. The beadstring may be rotated 180 degrees to show that $12+8=20$, but so does $8+12$.

## FRACTIONS

Beadstrings may be used to develop fraction understanding. Here are several fraction related ideas that you might like to try with older students using a 20,100 and 120 beadstring.

## FRACTIONS OF A QUANTITY

A beadstring may be used to support students calculating a fraction of a quantity. Finding fractions such as a half or a fourth (quarter) may be worked out by folding a string in half, or half and half again, in the case of calculating a fourth.

Later students can calculate:

- $1 / 5$ of a twenty beadstring.
- $\quad 1 / 4$ of a twenty beadstring.


## WHAT IS THE WHOLE?

Students will require three beadstrings on their desks ( 20,100 and 120 beads).

Ask the students to fold the strings in half and to note how many beads are in each half.

- 20 beadstrings: 10 beads
- 100 beadstrings: 50 beads
- 120 beadstrings: 60 beads.

Ask students to explain why one half is represented by different numbers of beads in each example. Students should come to the realisation that the size of the half depends on the size of the whole.

## EXTENDING THE THINKING

Show five beads and explain that the five beads represent half of the beads.

Ask how many beads in the whole? (10) (see Figure 9)


Figure 9. Five red beads represent half, therefore 10 represent the whole.

This could be extended to investigate if five beads represent one fourth of the beads, how many beads in one whole (20)(see Figure 10) or two wholes (40).


One fourth
-00000000000000000000
One fourth One fourth One fourth One fourth
Figure 10: One fourth is five beads, so one whole is 20 beads.

## PERCENTAGE: THE 100 BEADSTRING

The 100 beadstring is ideal for performing percentage calculations (literally: 'per hundred'). Students can work out tenths and hundredths of the hundred beadstring. Later, they can calculate halves, fourths, fifths of the 100 beadstring.

## THE 120 BEADSTRING

Add twenty beads on to the 100 beadstring to form a 120 beadstring. The 120 beadstring provides many more opportunities to calculate fractions of 120. Halves, thirds, fourths, fifths, sixths, eighths, tenths and twelfths of 120 may be calculated. Essentially when calculating one eighth of 120 beads the student could use division $120 \div 8$ to determine the number of beads required. In the case of three eighths, if a student can find $1 / 8$, the answer is essentially $3 \times 1 / 8$.

## LINKING FRACTIONS, DECIMALS AND PERCENTAGE

Working out $10 \%, 1 / 10$ and 0.1 of three different 100 beadstrings and laying them in three rows will highlight the links between $10 \%, 1 / 10$ and 0.1 .

## RENAMING THE VALUE OF BEADS

Consider all of the beads on the beadstring as representing a number.

If all beads on a twenty beadstring represents a total of two (2) then what value would a single bead represent? Ten beads would represent one (1) and therefore, one bead is one-tenth of one (0.1). (See Figure 11).
-00000000000000000000.
Figure 11: If twenty beads represents 2, then each bead is valued at 0.1.

Extending this idea, imagine the twenty beads represent four; what would a single bead represent ( 0.2 or $1 / 5$ or $2 / 10$ ). The number of beads can be changed and a variety of 'If... Then' statements can be used to encourage student thinking.
'If fifty beads represents ten... Then what does one bead represent?'

## CONCLUSION

This short article has demonstrated how versatile beads threaded on a string can be. They are not just a manipulative for young children, but can be used in quite sophisticated ways, as the last few fraction ideas suggest. To support teachers and schools trying to make the most of beadstrings, please refer to my free webinar available at www.youtube.com/drpaulswan.

## REFERENCES

Swan, P. (2020). Beadstring
Mathematics. Perth: A-Z Type
Beadstrings Webinar, www.youtube.com/drpaulswan

